**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Sol: Let’s find the probability that manager will meet his commitment:

Z = (X - *μ )/ σ* = (50 – 45) /8 = 5/8

P = 0.7340

Probability that manager cannot meet his commitment: 1-0.7340 = 0.266

1. The current age (in years) of 400 clerical employees is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol:

1. False: The SD is 6 and mean is 38 which means that 68% of the employees are between age 32 and 44 so the majority cannot be more than 44.
2. False. Z = (36-38)/6 , P = 37%, Probability of 36 employees attending the training is less.
3. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Sol: The difference between 2 *X*1 and *X*1 + *X*2 : Mean is same but Variance of 2X1 is double of X1+X2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Sol:

given: P(a<X<b) = 0.99, mean = 100 , sd = 20

we have to exclude 0.05 area from both the tails. Hence we find out Z score for 99.5 and 0.5:

Z(0.05) = -2.576 , Z(99.5) = +2.576

Z= (x-100)/20 => x= 20Z+100

a = -20\*2.576 + 100 = 48.5

b = 20\*2.576 + 100 =151.5

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Sol:

1. Let X be the profit from Division 1 and Y from Division 2. Then total profit of company will be Z = X+Y:

Z = 12 + 52

The 95% probability range for a normal distribution is typically defined as the range of values that lies within 1.96 standard deviations of the mean.

In this case, the mean is 12 and the standard deviation is 5. Therefore, we can calculate the 95% probability range for P as:

Lower value: 12 - (1.96 \* 5) = 2.2

In rupees: 99 million rupees

Upper value: 12 + (1.96 \* 5) = 21.8

In rupees: 981 million rupees

So, range = 99 – 981 million rupees

1. Calculate 5th Percentile:

For 5th percentile we need to find z score for probability of 0.05 which is -1.645

Therefore,

(Y - 12) / 5 = -1.645

Solving for Y, we get:

Y = -1.645 \* 5 + 12 = 3.775

3.775 \* 45 = 169.875 = Rs. 170 million

C. For Division1, we have:

P(Division1< 0) = P((X - 5)/3 < (0 - 5)/3)

= P(Z < -1.67)

= 0.0475

For Division2, we have:

P(Division2< 0) = P((X - 7)/4 < (0 - 7)/4)

= P(Z < -1.75)

= 0.0401

Division1 has a larger probability of making a loss in a given year.